Additional reinforcement in the masonry of historical buildings
tests, numerical analysis, practical application

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ABSTRACT: The contribution describes the problems of additional reinforcing of masonry structures. Two parts of this problem are solved: design of anchorage at masonry joints and design of the ultimate strength and applicability of a masonry structure. The experimental and theoretical analysis of the state of stress determination in the anchorage area, the anchorage length and the influence of some factors on these parameters are mentioned. The mechanisms of failure in masonry with additional reinforcement are described when the cross section is loaded by the eccentric force or with the bending moment. Based upon the experiments, a theoretical model for describing the behaviour of reinforced masonry beams has been developed taking into account the build up of the flexural stress to the point of failure. As examples of application of this system on historical objects in the Czech republic, the strengthening of burnt masonry and stone masonry Jezernice viaducts is described.

1 INTRODUCTION

Masonry from solid burnt bricks or stone bricks is one of traditionally used materials of walls and arches on historical buildings in the Czech republic. In consequence of movements of the basement or changes of stiffness during time, internal forces in these structures are redistributed, cracks appear and masonry needs maintenance. The strengthening of masonry structures with the Helifix system (additional rustproof steel reinforcement placed in slots filled with a special mortar, produced in Great Britain) is suitable and effective method. The most important advantage of this system is the fact that the reinforcement is inserted into the slots only in bed joints, which can be easily pointed. Therefore, the appearance of masonry is not affected at all. However, the design of this reinforcement is based mostly on the basis of empirical relations up to now. For design is convenient to solve two types of the problems: design of anchorage at masonry joints and design of the ultimate strength and applicability of a masonry structure.

2 REINFORCEMENT ANCHORAGE

2.1 Basic Analytical Relations for Reinforcement Anchorage

For displacement, deformation and stress determinations there can be written a simple differential equation based on a simplified 1D model of anchorage area

\[
\frac{E_v}{A_v} \frac{\partial^2 u_v(x)}{\partial x^2} - \frac{o}{2 \cdot (1+\mu_z) \cdot t} \cdot u_v(x) = 0, \tag{1}
\]

where \(E_v\) (\(E_s\)) = the modulus of elasticity of reinforcement (of grout), \(o\) = the circumference of reinforcement surface, \(A_v\) = the cross sectional area of reinforcement, \(t\) = the thickness of grout, and \(\mu_z\) = the Poisson’s coefficient of grout.
Solution for equation (1) is:
\[ u(x) = C_1 \cdot \exp(ax) + C_2 \cdot \exp(-ax), \]
where 
\[ a^2 = \frac{o \cdot E_z}{2 \cdot A_v \cdot E_t \cdot t \cdot (1 + \mu_z)}. \]

The normal stress in reinforcement and the shear stress in grout can be calculated from relations
\[ \sigma_v(x) = E_v \cdot a \cdot (C_1 \cdot \exp(ax) - C_2 \cdot \exp(-ax)), \quad \tau_{xz}(x) = \frac{E_z}{2 \cdot t \cdot (1 + \mu_z)} \cdot (C_1 \cdot \exp(ax) + C_2 \cdot \exp(-ax)). \]

The unknown integral constants \( C_1 \) and \( C_2 \) is possible to obtain from boundary conditions
a) Variant A
\[ s(v(x = 0) = \frac{P}{A_v}, \quad s(v(x = l) = 0 \quad (4) \]

The second boundary condition is based on assumption of known the length of the anchorage area \( l \).
b) Variant B
The first boundary condition will be identical to the variant A. At the end of the anchorage area following condition must be fulfilled:
\[ \tau_{xz}(x = l) = 0 \quad (5a) \]

This condition is equivalent - relating the formation of general solution - to the condition
\[ u_v(x = l) = 0 \quad (5b) \]

2.2 Numerical Model of Anchorage Area

2.2.1 Description of numerical model
The model of the anchorage area, which is used for the finite element method solution (FEM), is formed from the block of brick masonry in which is anchored the additional steel reinforcement - fig. 1.

Figure 1: Scheme of anchorage area of masonry by numerical modelling

The reinforcement is loaded with the constant force \( F = 10 \text{kN} \) on the front. On the frontal surface the masonry block and one part of grout mortar are supported by steel plate. The \( z \)-displacements on the frontal surface are equal zero. The brick of masonry at the upper and lower surface is supported – the displacements towards axis \( y \) are equal zero. The other surfaces are
without prescribed boundary conditions. The particular materials are modelled by means of spatial finite elements.

In the carried out calculations (numerical and analytical) there was considered with:

- Masonry as the composition material made from the burned solid bricks P 15 and mortar M 150 according to Czech standard CSN 73 1101. The masonry was modelled as the continuum with substitute stiffness. Following material characteristics were taken into account for the mathematical model there
  - the modulus of elasticity $E_{\text{def}} = 5.0 \text{ GPa}$,
  - the modulus of shear $G_{\text{def}} = 2.0 \text{ GPa}$,
  - Poisson’s coefficient calculated from the elasticity modulus $\mu_{\text{def}} = 0.25$,
  - the masonry strength under the main tension at joint failures $R_{\text{td}} = 0.12 \text{ MPa}$, at failures of masonry block building materials $R_{\text{td}} = 0.40 \text{ MPa}$,
  - the masonry strength under pressure $R_d = 2.4 \text{ MPa}$

- Grout mortar PP (double - component styrene resin Poly Plus, strength under pressure after four hours is 50 MPa, after 7 days 65 MPa, strength under tension after 1 day is 6 MPa, the elasticity modulus $E_{\text{pp}} = 25 \text{ GPa}$, the elasticity modulus under shear $G_{\text{pp}} = 10.5 \text{ MPa}$, Poisson’s coefficient $\mu_{\text{pp}} = 0.2$)

- Reinforcement H6 (HeliBar 6, the particular diameter is 6mm, the elasticity modulus determined according to tests - the average value $E_{\text{h6}} = 115 \text{ GPa}$, the cross sectional area 9.01 mm$^2$). The reinforcement is modelled in the cross section as the annulus.

- Reinforcement O6 (reinforcement 10425 (V) according to the CSN 731201, the diameter 6 mm, the cross sectional area of reinforcement 28.27 mm$^2$, the elasticity modulus $E_{\text{o6}} = 210 \text{ GPa}$, shear modulus $G = 81 \text{ GPa}$, Poisson’s coefficient $\mu_{\text{o6}} = 0.3$).

The calculations within the numerical model were carried out with assumption of physically linear and physically non-linear behaviour. More detailed specification of physically non-linear behaviour and modelling was published by Hladil (2000), Štepánek (2001).

Studies of problems within the anchorage length by means of mathematical modelling require the accomplishment of greater amount of sufficiently precise numerical calculations. At the method of finite elements the calculation accuracy depends especially on the sensitivity of model division and the type of used finite elements. Carried out numerical studies proved that:

- the state of stress near the reinforcement is substantially influenced by dimensions of solved block masonry cross sections that are smaller than 100/100 mm,
- the minimal applicable length of anchorage block is 200 mm.

### 2.2.2 Some results of numerical model

The state of stress in the anchorage area changes in dependence on

- the inlet dimensions in the steel slab in which the reinforcement passes through
- the arrangement of the anchored reinforcement in the masonry
- the reinforcement shape and material
- the grout shape and material
- the masonry material
- the length of the embedded reinforcement, i.e. on the length of anchorage $l$
- the boundary conditions (the way of supporting and loading)

With the described model was possible to obtain an influence of the above mentioned parameters on the results (stresses, displacements, anchorage length etc.). – see Štepánek (2001).

The influence of inlet size in the steel slab

Notation of solved variants is K-x, x = 3, 5, 7 and 11, x = diameter of inlet given as multiple of the anchored reinforcement diameter. Dependence of the force transferred into the masonry (grout mortar) in relation to the length from the beginning of anchorage is drown in Fig. 2 (Fig. 3).

The Influence of Grout Location inside the Anchorage Block

The influence of reinforcement location on the state of stress of tested block in the vertical direction was tested in the anchorage block of cross section dimensions 300/450 mm. The distance of reinforcement mass centre from the vertical block surface is 25 mm, the distance of reinforce-
ment mass centre from the low horizontal block surface is \( h = 225, 300 \) and \( 375 \) mm. For each material (reinforcement, grout, masonry) the percentage force difference (relating to particular material) was expressed in selected cross sections of the anchorage block according the relation:

\[
\text{difference} = \left( \frac{F_{M225} - F_{M375}}{F_{M225,poc}} \right) \times 100 ,
\]

where \( F_{M225} \) = force in given material for model \( h = 225 \) mm in cross section \( z \), \( F_{M375} \) = force in given material for model \( h = 375 \) mm in cross section \( z \), \( F_{M225,poc} \) = force in given material for model \( h = 225 \) mm at the beginning of anchorage area (\( z = 0 \)).

The influence of the vertical reinforcement location in the anchorage block relating to state of stress in particular materials for the models \( h = 225 \) and \( 375 \) mm is given in Fig. 4.

![Figure 2: Force in masonry](image)

2.3 Testing

Tests of anchorage reinforcement have been carried out on six experimental objects (anchorage blocks). Further tests are being prepared. Three blocks were reinforced with concrete reinforcement V6 (notation according to the standard CSN 73 1201); three blocks were reinforced with rust-resistant reinforcement Helifix. The lengths of anchorage reinforcement inside the blocks were at particular experimental objects 600, 450 and 300 mm. In the block was always only one reinforcement bar. The notation of particular experimental objects is as follows: X-l, where X indicates the reinforcement type (\( X = b \) for concrete reinforcement, \( X = h \) for rust resistant reinforcement), \( l \) is the length of reinforcement anchorage in mortar (\( l = 300, 450 \) and \( 600 \) mm).

Blocks with reinforcement V were not clamped in vertical direction; the second trio was clamped with joiner clips - the upper surface with steel slab against the lower board made from steel slab. The following parameters were measured – fig. 5:

- change of width along the horizontal and vertical masonry gaps
- displacement of reinforcement within the rod length
- displacement of the end of the reinforcing bar
The scheme of tests arrangement and diagrams of measured values are presented in Fig. 5 and Fig. 6.

Table 1 presents both the absolute values of anchorage lengths and the relative values of anchorage lengths in relation to the diameter of used reinforcement for various Young modulus of grout which were found out by analytical, numerical and experimental tests. In results there was not comprised the enlargement of anchorage lengths as a result of safety access as this was introduced e.g. by EC. The relation of the anchorage length in dependence on the reinforcement diameter was found out within mathematical models.
3 STRENGTHENING OF MASONRY BEAMS

Strengthening of a structure can be effectively achieved by the insertion of prestressed or non-prestressed reinforcement into the areas of the structure under tension. At present the design of retrofit strengthening of the masonry is based mostly upon empirical data and up today, a comprehensive design algorithm has not been developed. The aim of the test programme was

• to document the suitability and efficiency of retrofitted masonry structures with non-prestressed reinforcement,
• to provide basic data for the development of a design theory for strengthening.

Table 1: Anchorage lengths

<table>
<thead>
<tr>
<th>Reinforcement type</th>
<th>Modulus of grout elasticity</th>
<th>Particular diameter of reinforcement (mm)</th>
<th>absolute anchorage length (mm)</th>
<th>anchorage length as multiple of reinforcement diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Numerical solution</td>
<td>Analytical solution</td>
</tr>
<tr>
<td>Heli fix</td>
<td>7,8</td>
<td>6,0</td>
<td>155</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td>22,0</td>
<td>6,0</td>
<td>130</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>30,0</td>
<td>6,0</td>
<td>115</td>
<td>66</td>
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<tr>
<td>Reinforcement</td>
<td>7,8</td>
<td>6,0</td>
<td>385</td>
<td>312</td>
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<tr>
<td></td>
<td>22,0</td>
<td>6,0</td>
<td>242</td>
<td>213</td>
</tr>
<tr>
<td></td>
<td>30,0</td>
<td>6,0</td>
<td>165</td>
<td>106</td>
</tr>
</tbody>
</table>

Figure 6: Relation between displacement of free end of reinforcement and acting force
3.1 Testing program and results

The testing programme comprised two sets of masonry beams of overall dimensions: set A 300/600-2700mm, set B 450/600-2700mm. Each set of tests consisted of 10 beams reinforced with the types according Fig. 7a

- Type 1 (3 samples): Reinforcement in the upper and lower bed joint placed symmetrically,
- Type 2 (3 samples): Reinforcement in the lower bed joint only, placed symmetrically,
- Type 3 (3 samples): Reinforcement in the upper and lower bed joint on one side of the beam,
- Type 0 (1 sample): An un-reinforced beam for comparison.

In the following text pictures we will use notation of beams in the form X-n, where X∈ {A, B}, n∈ {0, 1, 2, 3} – number of reinforcing types.

The beams were loaded by the vertical forces applied at 1/3 and 2/3 of the span - fig. 7b. For the purpose of obtaining the most accurate model of boundary conditions during the course of the loading of the beam in a real masonry structure, stiff steel plates were placed on the end faces of the beams on a bed of gypsum mortar. The two faceplates were connected with steel tendons – Fig. 7c,d. The nuts of the tendons were only finger tight so no significant tensile stress was induced in the tendons. The layout of the loading tests and the measuring equipment is shown in fig. 7.

Figure 7: Scheme of tested masonry beams

In Fig. 8 the measured results for the different types of reinforcement layout are presented:

- The deflection of the beams at mid span W_{L/2} and the loading force F,
- The average sum of the axial forces in the bottom of external tendons N and the loading force F.

The average results of each type of reinforcing are shown in the diagrams. The upper two diagrams refer to the beams in set A, the bottom two refer to the beams in set B. The results of the measurements performed on the beams without reinforcement A-0, B-0 are given for comparison.
3.2 Analytical model of tested beams behaviour

Description of the theoretical mode of behaviour of the tested beams will be given during loading presuming the beam fails in flexion. Further experiments are needed to investigate the behaviour of members under the shear load. To study the bending stress, an ideal mode of behaviour of the reinforced masonry beam develops in three phases, which are characterised by different stiffness:

- **1\(^{st}\) Phase:** Up to cracks onset in the brickwork all materials act elastically according to Hook’s Law. The flexural stiffness defined, presumes the complete utilization of the cross section, taking into account the elastic behaviour of all the material. The ultimate load of the cracked masonry shall be estimated from the tensile strength of the section of the masonry continuum in tension.

- **2\(^{nd}\) Phase:** With the progressive development of cracks in the masonry the flexural stiffness decreases, however the reinforcement (rebars and infill mortar with relatively high bond strength in the masonry) continue to behave elastically. The stiffness is defined as the stiffness of the ideal cross-section without tensioned masonry.

- **3\(^{rd}\) Phase:** The flexural stiffness of the beam rapidly decreases after cracking is set up in the infill mortar and the tensile stresses are distributed by the rebars only.
The impact of the actual forces in the tendons has to be isolated from the empirical experiments. Considering that the beams in both sets A and B differ only in the width (the total amount of reinforcement in both cases is the same) so it is evident that the model of behaviour determined by the defined presumptions will vary only insignificantly with the different types of reinforcement acting in the 2nd and 3rd phases. However, the results of the test show that the beams in set B have significantly greater stiffness and the failure occurred at essentially higher levels of the load. This fact is explained by the simplified consideration of the masonry as a substitute continuum, i.e. the constant cross-section parameters are considered through the whole length of the member. Due to the brick bond in the masonry several different types of cross-section exist in the real specimen with various ratios of individual basic materials (original mortar, bricks). The number of different cross-sections depends on the complexity of the brick bond. Some cross-sections cut only the layers of brick and bed joints, others also cut a certain number of vertical joints. It can be seen that the cross-sections along the member length in the 2nd and 3rd phases of the idealized model, interact. During the test, hairline cracking was observed in certain cross-sections in the connection between the vertical mortar joints, however due to the brick bond the connected bricks helped to distribute the stress. A theoretical model taking into account this kind of behaviour shown in the tested members is relatively complex. At this stage there is not enough data for its formulation and it will be the subject of future research.

The simplified model of behaviour defined for the bending failure for the reinforced beams (width ‘b’ [mm], rebars HeliBar and the infill mortar HeliBond MM2 in the slots at the first and last horizontal bed joint, placed symmetrically), considering a classic brick bond is built up with the following steps:
- Calculation of the stiffness in individual phases for a beam of a width 300 mm,
- Calculation of the load-bearing capacity of a beam of a width 300 mm for determination of the interface between the individual phases,
- Extrapolation of a working diagram of a beam of a width 300 mm on a beam of width ‘b’ (×b/300).

For determination of the ultimate load-bearing capacity we recommend the following limits:
- Strain of additional tensile reinforcement – the experiments show this should be suitable in the range from 0.60 – 0.80%,
- Maximum allowable ratio $M_{load-bearing capacity, strengthened}/M_{load-bearing capacity, non-strengthened} \leq k$, where coefficient $k > 1$ will be evolved from realised and future experiments.

The next restriction for the use of additional reinforcement of masonry members under bending is based upon the deformation of a member. The stiffness in separate phases of the idealised behaviour of a member maybe used to define the deformation in accordance with the previous test. For the design itself it is again necessary to provide reliability coefficients to allow for the reduction of the stiffness under any imposed load.

In the case of asymmetric reinforcement, the described models do not give satisfactory results. Results based on this model give lower values of the load bearing capacity; the model gives essentially lower stiffness. Since asymmetrically reinforced beams are also stressed in the torsion the strength of the masonry is highly dependent on the brick bond. Without further experiments and the associated detailed theoretical analysis, it is not possible to create a suitable model for the behaviour for members reinforced asymmetrically.

4 REPAIR OF VIADUCT IN JEZERNICE

The viaduct in Jezernice is an arch bridge of total length 399 m. 30 arches form the structure. Rise of arches is 4.5, 5.1, and 5.7 m, ground plan of the piers is 2.6/10.8 m and height of the piers is 5.5 – 11.0 m. Total height of the bridge is 18.5 m. Essentially, the structure consist of two parallel continuously connected bridges consecutive constructed. The stone piers with masonry arches form first bridge and the second one is formed by the stone piers and the concrete shells.

Prior to the repair, following faults were diagnosed:
- Leaches on the soffits and piers caused by damaged insulation.
Cracks in the subgrade caused by freezing and thawing of moisture in the masonry surface layers.

• Vertical cracks in the piers and tendency of material separation at the corners.

• Concrete of the arches was carbonized.

• Cracks between the parapet wall and arches.

• Vertical cracks in the parapet walls.

• Carbonated concrete and corroded reinforcement in the cornices.

The repair was carried out in the following steps:

• Improvement of the foundation structures reliability (construction of the micro-piles, extension of the foundation and the crack injection).

• Strengthening of the origin piers disrupted by the cracks (two rods HeliBar 6 were embedded into the cut slots filled with mortar HeliBond MM2. The cracking areas and disrupted concrete were deeply re-grouted). 40000 meters of reinforcement HeliBar 6 were applied including adjacent viaduct.

• Repair of the disrupted concrete structures (abrasive blasting).

SUDOP Prague made design of whole reconstruction; investor was Czech Railways, general supplier was Dopravní stavby Brno, built 2000 – 2001.

5 CONCLUSION

From the tests of anchorage blocks and in comparison with experimentally found out relations for the concrete reinforcement and the reinforcement HELIFIX there is obvious that:

• the anchorage length of one bar HELIFIX reinforcement is smaller than 300 mm, the anchorage length of concrete reinforcement is higher than 300 mm

• the anchorage of HELIFIX reinforcement in lengths 300 mm and higher failures at reinforcement tensions which are more than 700 MPa, at concrete reinforcement there was the stress at anchorage failures about 80 - 90 Mpa.

On the basis of 20 tests of masonry beams, a theoretical model of behaviour can be evolved for members loaded in bending. In the case of beams reinforced symmetrically relative to the longitudinal vertical plane of symmetry of a beam, the ideal model follows the course of behaviour during the test. The object of the detailed exploration will be mainly concentrating on the behaviour of the beams with asymmetric reinforcement. With a view to setting-up a suitable design algorithm, derived coefficients reducing the parameters of behaviour of retrofit reinforced masonry from the viewpoint of required reliability will be established according to design standards. The small range of experiments performed to date does not allow these coefficients to be determined reliably. The object of additional experimental and theoretical research will be the investigation of shear forces on the behaviour of the members fitted with a retrofit reinforcement, which may be quite restrictive in the case of slender beams.

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